A novel way to obtain fixed point functions using Sine Cosine Algorithm

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ABSTRACT

In this paper, we introduce a new iterative method to finding the fixed point of a nonlinear function. In fact, we want to offer a new way to obtain the fixed point of various functions using the Sine Cosine Algorithm. This method is new and very efficient for solving a nonlinear equation. We explain this method with three benchmark functions and compare results with others methods, such as ALO, MVO, SSA.

KEYWORDS: meta-heuristic algorithms, Fixed point problems, Sine Cosine Algorithm.

1 Introduction

Obtaining the roots of equations, especially nonlinear equations, is one of the most important topics in engineering and basic sciences. For this sake, many researchers have checked this problem for some years [34,42].

The Bisection method is one of the most important methods in numerical calculations to find the root of a continuous function, which we know has a different sign at two points. This method is one of the simplest ways to find the root of a function in numerical calculations.

Meta-heuristic or meta-heuristic or meta-heuristic algorithms are a type of random algorithms that are used to find the optimal answer. Optimization methods and algorithms are divided into two categories: exact algorithms and approximate algorithms.

Well-known population- based meta-heuristic algorithms include evolutionary algorithms (genetic algorithm) [2], ant colony optimization (ACO) [3, 4], bee colony(BC) [5], particle swarm optimization method (PSO) [6], forest optimization algorithm (FO) [7], Battle royale optimization algorithm (BRO) [8], runner- root algorithm(RRA) [9] , intelligent water drops algorithm (IWD) [10], Artificial Bee Colony algorithm(ABC) [11, 12], Firefly Algorithm(FA) [13] , Differential evolution (DE) algorithms [14], biogeography based optimization (BBO) algorithm [15].

In recent years, new meta-heuristic algorithms have been developed with respect to living organisms in nature (inspired by nature), the most famous of which are the Gray Wolf Optimization Algorithm(GWO) [16], the Dragonfly algorithm (DA) [17], the Flower Pollination Optimization Algorithm (FPA) [18], Whale optimization Algorithm (WOA) [19], Grasshopper Optimisation Algorithm (GOA) [20], social spider algorithm (SSA) [21], Sine Cosine Algorithm (SCA) [22], Multi-Verse Optimizer algorithm (MVO) [23], Moth-flame optimization algorithm (MFO) [24], Ant Lion Optimizer algorithm (ALO) [25], Emperor Penguins Colony algorithm [26] and so on [1,44], [27-33].
In this paper, we introduce a novel iterative method that obtain the fixed point of various functions using the Sine Cosine Algorithm.
In Sect. 2, the Sine Cosine Algorithm is explained and fixed point problem is illustrated. Also suggested method illustrated in Sect.3. Section 4 measures the resolution of the offered method by different methods on several functions. Also, the result is available at Sect. 5.

2 Preliminaries

In the present section, the Bisection method and the Sine Cosine Algorithm is explained and fixed point problem is illustrated.

2.1 The Sine Cosine Algorithm (SCA)

The Sine Cosine algorithm, or in other words, the Sin Cos algorithm, abbreviated SCA, is a metaheuristic or meta-heuristic algorithm. This algorithm was presented by Seyedali Mirjalili in 2016 in a paper entitled SCA: A Sine Cosine Algorithm for Solving Optimization Problems in the journal Knowledge-Based Systems of Elsevier. In the following, the operation and training and night code of this optimization algorithm will be introduced.
In general, population-based optimization methods start the optimization process with a set of random solutions. This random set is evaluated again and again by a target function and is improved by a set of rules that is the core of the optimization method. Because population-based optimization techniques randomly seek to optimize optimization problems, there is no guarantee of finding a solution in a single period. However, with a sufficient number of random solutions and optimization (iteration) steps, the probability of finding a global optimal increases.
Regardless of the differences between algorithms in population-based optimization, the optimization process is usually divided into two stages:

- Exploration Phase
- Exploitation Phase

An optimization algorithm in the exploration phase combines random solutions with a high rate of crashes to find promising areas of search space. In the exploitation phase, however, gradual changes in stochastic solutions occur, and the stochastic changes are significantly less than the existing phases in exploration.

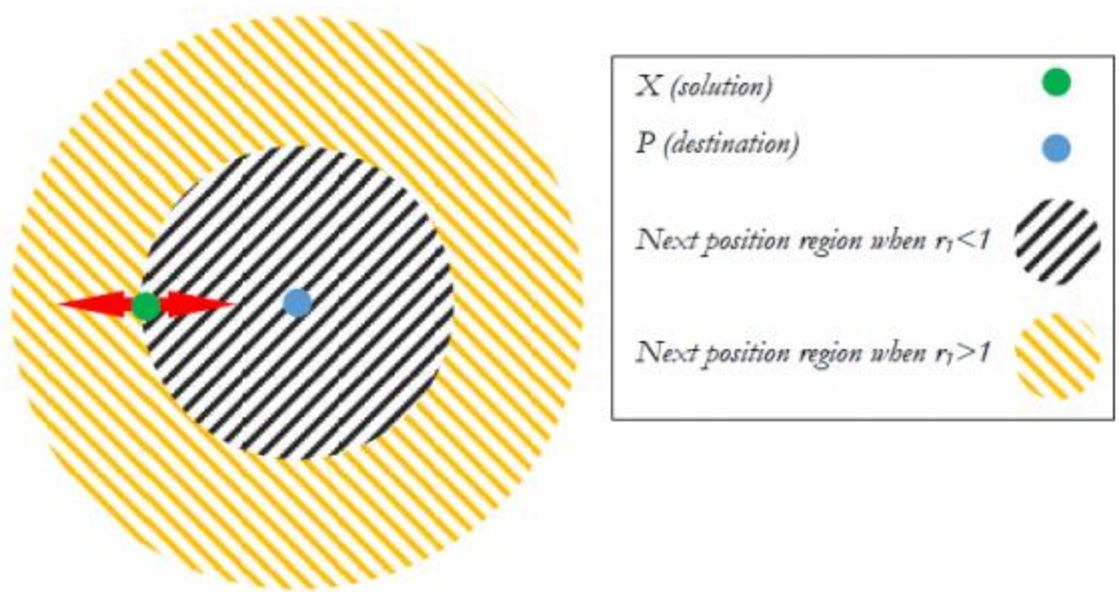


Figure 1. Effects of Sine and Cosine

The figure above shows how the SCA algorithm equations determine the space between two solutions in the search space. It should be noted that this equation can be extended to higher dimensions, although a two-dimensional model is shown in the figure above.
The cyclic pattern of sine and cosine operation allows one answer to be placed around another again. This can ensure the utilization of the defined space between the two solutions. To explore the search space, solutions must also be able to search outside the space between the relevant destination. This can be achieved by changing the function range of the sine and cosine, as shown in the figure below.

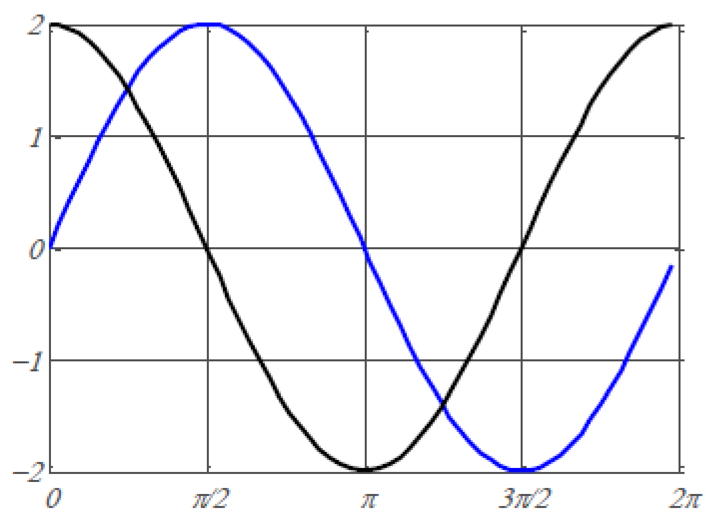


Figure 2. Sine and cosine with range of [-2,2]

A conceptual model of the effects of sine and cosine with amplitudes in [-2, 2] is shown in the figure below. The space between the answer itself and the other solution.

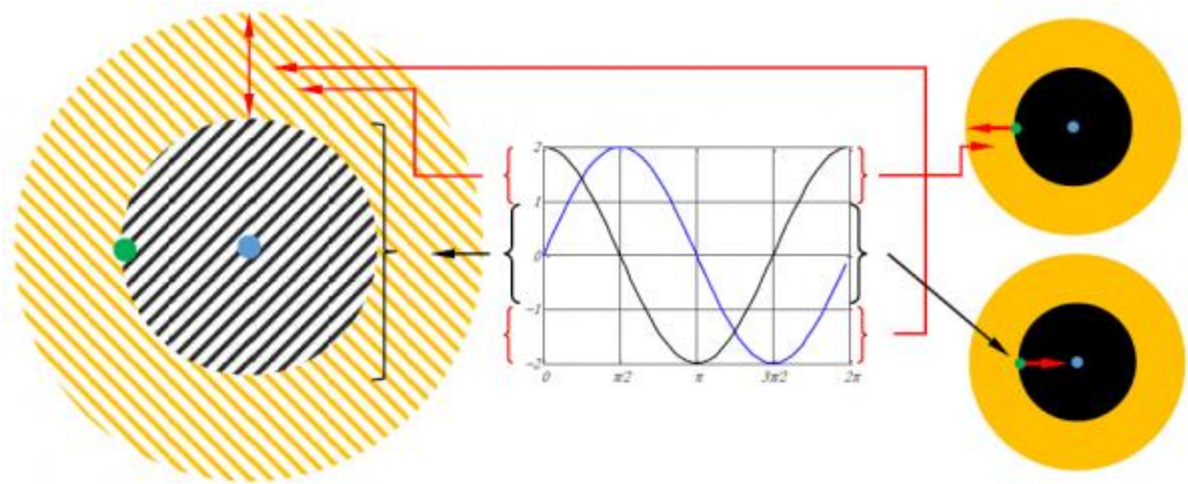


Figure 3. Sine and cosine with the range in [-2,2] allow a solution to go around (inside the space between them) or beyond (outside the space between them) the destination

An algorithm must be able to balance exploration and exploitation to find promising areas of search space and eventually converge to a desirable global answer. The pseudo-code of the SCA algorithm is presented in the following figure:

```
Initialize a set of search agents (solutions)(X)
Do
   Evaluate each of the search agents by the objective function
   Update the best solution obtained so far (P=X*)
   Update r1, r2, r3, and r4
   Update the position of search agents using Eq. (3.3)
While(t< maximum number of iterations)
Return the best solution obtained so far as the global optimum
```

Figure 4. General steps of the SCA Algorithm

The SCA algorithm starts the optimization process with a set of random solutions. The algorithm then stores the best solutions obtained, assigns them as destinations, and updates other solutions accordingly. Meanwhile, the range of sine and cosine functions has been updated to emphasize the utilization of search space by increasing the number of iterations. The SCA algorithm terminates the optimization process when the iteration counter is higher than the maximum number of iterations by default. However, any other termination conditions can be considered, including the maximum number of performance evaluations or the optimal general accuracy obtained.

With the above operators, the SCA algorithm is theoretically able to determine the global optimization for the following reasons:

1. The SCA algorithm creates and improves a set of random solutions to a given problem, so it inherently benefits from high exploration and optimal local avoidance compared to other solution-based algorithms.

2. When sine and cosine functions return values greater than 1 or less than -1, different areas of the search space are explored.

3. When the sine and cosine return values are between -1 and 1, the promising areas of the search space are explored.
4. SCA algorithm is easily transferred from exploration to operation by using adaptive amplitude change in sine and cosine functions.
5. The best global optimal approximation is stored in variables as a destination and never disappears during optimization.
6. Because solutions are always updating their positions on the best solution ever achieved, there is a tendency for the best areas of search spaces when optimizing.
7. Since the SCA algorithm considers the optimization problem as black boxes, it can easily be used in various fields provided the appropriate formulation.

2.2 Definition the fixed point

In mathematics, a fixed point (invariant point) of a function isa point that is mapped to itself by the function. In other word, a number c is a fixed point for a given function g if g(c) = c. A set of fixed points is sometimes called a fixed set. An iterative method for solving equation g(x) = x is the recursive relation xi+1= g(xi), i = 0, 1, 2, . . ., with some initial guess x0. The algorithm stops when one of the following stopping criterion is met:
•D1: total number of iterations is N, for some N, fixed a priori.
•D2: |xi+1− xi| < € for some €, fixed a priori.
This procedure is shown in figure 5.

Repeat

1. Give the transcendental equation $x = g(x)$,
2. Give initial guess in interval $[a, \quad b]$,
3. Do $x_{i+1} = g(x_i)$

until (D_1 or D_2 is true)

Figure 5: Fixed Point iteration scheme

3 The Sine Cosine Algorithm for solving fixed point of functions

At present part, we present one modern repetitious procedure to gain the solution estimation of a fixed point question as g(x) = x. We describe a function f(x) = g(x) - x. Accordingly the question of discovering the fixed points of g(x) is decreased to discovering the roots of f(x). We subsequent describe a function h(x) =|f(x)|. The question of discovering the roots of f(x) is better decreased to discovering an x that minimizes h(x). The opinion where is that for obtaining the half point of the distance I to begin with one volunteer answer, SCA algorithm is utilized to impute one superior estimation and determined a distance Ik = [ak , bk] one volunteer solution xk is calculated utilizing the SCA algorithm. If f(xk) = 0 we are accomplished, again calculate one modern distance Ik+1 into Ik pertaining against whether f(xk). f(ak) < 0 or f(xk). f(bk) < 0.

4 Implement methods on various functions

In this section, we illustrate our algorithm with some examples and compare the results with other evolutionary optimization algorithms such as ALO, MVO, SSA .

4.1 Introducing different functions

Introducing different functions

$$g_1(x): \frac{x^2}{4000} - \cos(x) + 1 = x \qquad ; \qquad x \in [-20, +20]$$

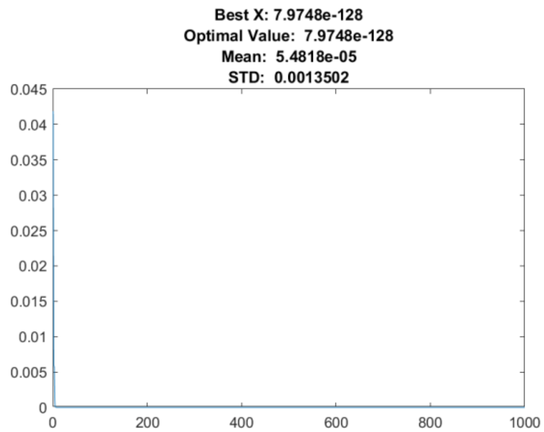
$$g_2(x): 10 + x^2 - 10\cos(2\pi x) = x \qquad ; \qquad x \in [-20, 1)$$

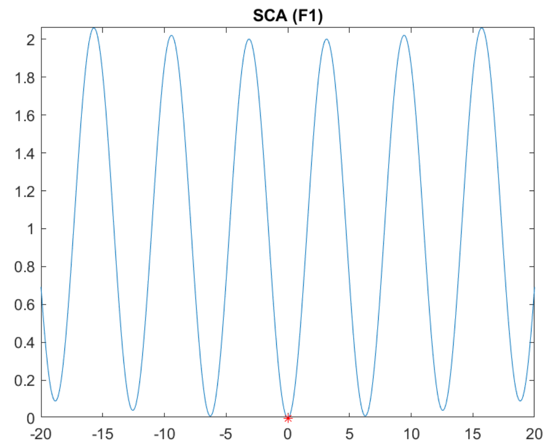
$$g_3(x): 20 + e - 20e^{-0.2\sqrt{x^2}} - e^{\cos(2\pi x)} = x \qquad ; \qquad x \in [1, 21]$$

Results for the three functions are shown in Table 1 and Their diagrams are also shown in Figures 6-8. Figures 6 to 8 show Diagram of the recovery process of the g1 to g3 functions by the SCA algorithm in (a) and diagram of the finding of the fixed point of the g1 to g3 functions by the SCA algorithm in (b).

Table 1 The Comparative results obtained for each function by ALO, MVO, SSA and SCA algorithms

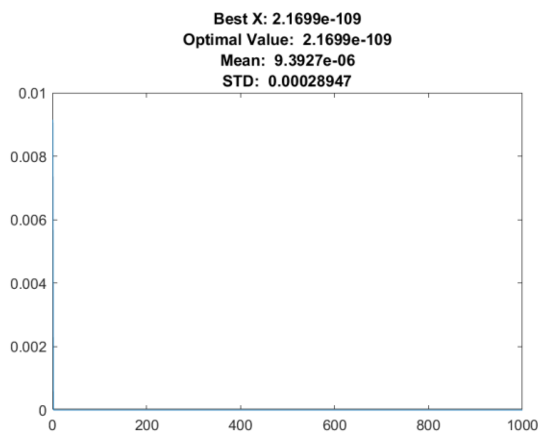
algorithm	Components	g1(x)	g2(x)	g3(x)
ALO	error	1.49E-10	5.06E-10	1.22E-09
	X_best	1.49E-10	5.04E-03	2.08E+01
	mean(e)	1.01E-04	1.91E-06	8.57E-05
	std(e)	2.83E-04	4.69E-05	1.44E-03
MVO	error	1.26E-07	5.33E-07	1.70E-05
	X_best	1.26E-07	5.04E-03	2.01E+01
	mean(e)	2.13E-04	1.89E-06	1.40E-03
	std(e)	1.17E-03	3.24E-05	3.59E-03
SSA	error	9.38E-12	2.54E-10	8.67E-09
	X_best	-9.38E-12	0.005040943	19.92296745
	mean(e)	9.69E-05	1.82E-06	0.004716755
	std(e)	0.000929696	3.62E-05	0.008069696
SCA	error	7.97E-128	2.17E-109	5.60E-05
	X_best	7.97E-128	2.17E-109	1.99E+01
	mean(e)	5.48E-05	9.39E-06	2.98E-04
	std(e)	1.35E-03	0.000289475	1.38E-03

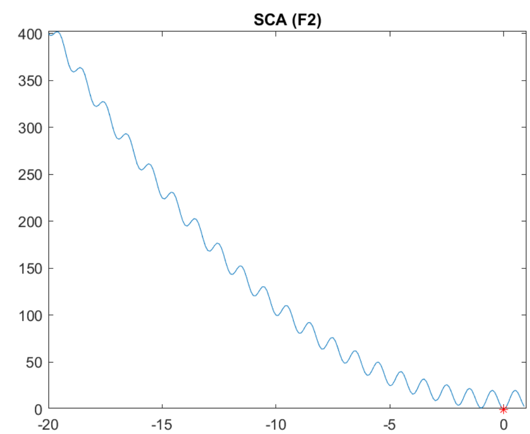




(a) (b)

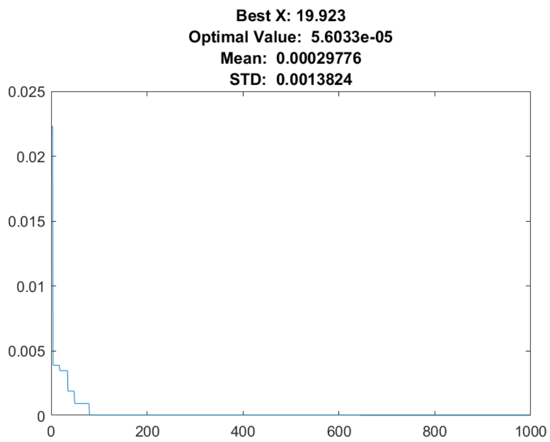
Figure *6*: Diagram of the recovery process of the g1 function by the SCA algorithm in (a) and diagram of the finding of the fixed point of the g1 function by the SCA algorithm using the intersection of the diagram g1 (x) = x in (b)





(a) (b)

Figure *7*: : Diagram of the recovery process of the g1 function by the SCA algorithm in (a) and diagram of the finding of the fixed point of the g2 function by the SCA algorithm using the intersection of the diagram g2 (x) = x in (b)



(a) (b)

Figure *8*: : Diagram of the recovery process of the g1 function by the SCA algorithm in (a) and diagram of the finding of the fixed point of the g3 function by the SCA algorithm using the intersection of the diagram g3 (x) = x in (b)

5 conclusion

In this paper, we introduce a novel iterative method for finding a fixed point of a function g in a real interval $[a, b] \subseteq R$ by using the Sine Cosine algorithm. If the function g is hard, it is sometimes difficult to determine suitable initial value close to the location of a fixed point. Derivative method (find the derivative of $g(x) - x$ and find its root) is also sometimes not useful for various reasons like the derivative may not exist, the derivative is hard to compute or finding the root of a derivative itself may be difficult. SCA algorithm helps in finding a good initial value and the proposed method does away with the need to compute the derivative. Our proposed algorithm is easy to use and reliable. As comparison with other algorithm shows, the accuracy of our proposed method also is good.

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